Math 312

Implicit Function Theorem Supplementary Problems

1. If $ye^{-xy} = \sin x$ find $\dfrac{dy}{dx}$ when it exists.

2. In each case find y' and y''

   (a) $xy - y^3 = 1$, (b) $xye^y = 10$, (c) $y + y^2 + y^3 = x$.

3. Find z_x and z_y when

   (a) $xe^y = z^2 \sin z$, (b) $x + y + z - \sin(xyz) = 0$

4. Show that $e^z(x^2 + y^2 + z^2) - \sqrt{1 + z^2} + y = 0$ defines $z = z(x, y)$ at $(x, y, z) = (1, 0, 0)$. Approximate z by its tangent plane at this point.

5. Does the equation $(x^2 + y^2 + z^2)^{1/2} - \cos z = 0$ define $y = y(x, z)$ in the neighborhood of $(0, 1, 0)$? What about $z = z(x, y)$?

6. If $u = f(x - u, y - v)$ find $\left(\dfrac{\partial u}{\partial y}\right)_x$.

7. Given $u^3 + xv = y$, $v^3 + yu = x$ find $\left(\dfrac{\partial u}{\partial x}\right)_y$

8. Given $xu^2 + yv^3 = xy^2$, $yu^3 - xv^2 = x^2y$ find $\left(\dfrac{\partial v}{\partial y}\right)_x$

9. If $xu + uv - v = x^3$, $v^2 - xv + u = x^2$ find $\dfrac{du}{dx}$

10. When can the equations $x = u + v$, $y = u^2 + v^2$ be inverted? Evaluate $\dfrac{\partial v}{\partial x}$.

11. If $x = u + v + w$, $y = u^2 + v^2 + w^2$, $z = u^3 + v^3 + w^3$ show that $\left(\dfrac{\partial u}{\partial x}\right)_{yz} = \frac{vw(w-v)}{(u-v)(v-w)(w-u)}$ and find $\left(\dfrac{\partial u}{\partial y}\right)_{xz}$ and $\left(\dfrac{\partial u}{\partial z}\right)_{xy}$.

12. Verify that $\dfrac{\partial(X, Y)}{\partial(r, s)} = \dfrac{\partial(x, y)}{\partial(u, v)} \dfrac{\partial(u, v)}{\partial(r, s)}$ when $x = u\cos v$, $y = u\sin v$, $u = r + s$, $v = r^2 + s^2$ and $x = X(r, s)$, $y = Y(r, s)$.